Study Guide

Division of Polynominals
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Polynomials: Division
A monomial is the product of a number and an unknown variable or unknown variables. 6xy is a monomial. The sum of two or more monomials is called a polynomial. Here is an example of a polynomial: $y^2 + 4y + 3$.
A binomial is a polynomial with exactly two monomial terms. 3x + 4 is a binomial. A trinomial is a polynomial with exactly three terms. 4xy - 3x +6y is a trinomial.

Before dividing polynomials, recall the following properties associated with exponents:

Exponential Properties for Division

$$\frac{a^m}{a^n} = a^{m-n}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^0 = 1$$

Example 1: Divide.

$$\frac{12x^3y}{-3xy}$$

(1) $\frac{12}{-3} = -4$ **(2)** $\frac{x^3}{x} = x^{3-1} = x^2$ **(3)** $\frac{y}{y} = y^{1-1} = y^0 = 1$ **(4)** $\frac{(-4)(x^2)(1)}{-4x^2}$

Step 1: Divide the whole numbers: 12 ÷ -3 = -4.
Step 2: Use the properties above to divide the variables. Begin with the x-variables. x-cubed divided by x equals x-squared.
Step 3: Now divide the y-variables. y divided by y equals y to the power of zero. Any number taken to the power of zero equals 1.
Step 4: Finally, multiply the quotients back together.

The answer is $-4x^2$.
Dividing a Polynomial by a Monomial:

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial. Then, combine the similar terms.

Example 2: Divide.

$$\frac{3m - 9n}{3}$$

(1) $\frac{3m}{3} = m$ **(2)** $\frac{-9n}{3} = -3n$ **(3)** $m - 3n$

Step 1: Divide 3m by 3, to get m.
Step 2: Divide -9n by 3, to get -3n.
Step 3: Combine the terms.

Answer: m - 3n

Dividing a Polynomial by a Polynomial:

Dividing one polynomial by another is very similar to long division.

Example 3: Divide $(6x^2+8x+8)$ by $(3x+1)$.

Step 1: $3x+1\overline{)6x^2+8x+8}$

Step 2: $3x+1\overline{)6x^2+8x+8}$ $\quad 2x+2$

Step 3: $\quad -(6x^2+2x)$

Step 4: $\quad 6x+8$

Step 5: $\quad -(6x+2)$

$\quad 6$

Step 1: Write the problem as a long division problem. The binomial belongs on the outside of the division symbol because it is the term we are dividing by.

Step 2: Now, we can begin dividing.

$(3x)(2x) = 6x^2$ So, 2x belongs above the 8x.

Step 3: The next step is to multiply 2x by (3x + 1).

$(2x)(3x+1) = 6x^2 + 2x$ Subtract that product from $6x^2 + 8x$. Now, bring the + 8 straight down beside the 6x.

Step 4: (3x)(2) = 6x, so we place the 2 above the 8 in the answer.

Step 5: Multiply 2 by (3x + 1) to get 6x + 2. Subtract (6x + 2) from (6x + 8). There is a remainder of 6, so we write the remainder as a fraction with the binomial as the denominator.

Answer: $2x+2+\frac{6}{3x+1}$